MASSIVE ELEMENTARY PARTICLES AND BLACK HOLES IN RESUMMED QUANTUM GRAVITY

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We use exact results in a new approach to quantum gravity to show that the classical conclusion that a massive elementary point particle is a black hole is obviated by quantum loop effects. Further phenomenological implications are discussed.

1 Introduction

Albert Einstein showed that Newton's law, one of the most basic laws in physics, is a special case of the solutions of the classical field equations of his general theory of relativity. Specifically, $g_{00} = 1 + 2\Phi_N \Rightarrow \nabla^2\Phi_N = 4\pi G_N \rho$ from $R^{\alpha\gamma} - \frac{1}{2}g^{\alpha\gamma}R = -8\pi G_N T^{\alpha\gamma}$, etc., where he have now introduced the familiar metric of space-time $g_{\mu\nu}$, the Newtonian potential Φ_N, Newton's constant G_N, the mass density ρ, the contracted Riemann tensor $R^{\alpha\gamma}$, and the appropriate energy momentum tensor $T^{\alpha\gamma}$. There have been several successful tests of Einstein's theory in classical physics [1–3].

Heisenberg and Schroedinger, following Bohr, formulated a quantum mechanics that has explained, in the Standard Model(SM) [4], all established experimentally accessible quantum phenomena except the quantum treatment of Newton's law. Indeed, even with tremendous progress in quantum field theory, superstrings [5, 6], loop quantum gravity [7], etc., no satisfactory treatment of the quantum mechanics of Newton's law is known to be correct phenomenologically. Here, we apply a new approach [8] to quantum gravitational phenomena, building on previous work by Feynman [9, 10] to get a minimal union of Bohr's and Einstein's ideas.

There are four approaches [11] to the attendant bad UV behavior of quantum gravity (QG): extended theories of gravitation such as supersymmetric theories - superstrings and loop quantum gravity; resummation, a new version of which we discuss presently; composite gravitons; and, asymptotic safety – fixed point theory, recently pursued with success in Refs. [12, 13]. Our approach allows us to make contact with both the extended theory approach and the asymptotic safety approach.

Our new approach , resummed quantum gravity, is based on well-tested YFS [14, 15] methods. We first review Feynman's formulation of Einstein's theory in Sect. 2. We present resummed QG in Sect. 3. In Sect. 4 we discuss Newton's law. In Sect. 5 we discuss the black hole physics, some of which is related to Hawking radiation [16].

2 Review of Feynman's Formulation of Einstein's Theory

For the known world, we have the generally covariant Lagrangian

$$\mathcal{L}(x) = -\frac{1}{2\kappa^2}\sqrt{-g}R + \sqrt{-g}L^{\mathcal{G}}_{SM}(x) \qquad (1)$$

where R is the curvature scalar, $-g = -\det g_{\mu\nu}$, $\kappa = \sqrt{8\pi G_N} \equiv \sqrt{8\pi/M^2_{Pl}}$, where G_N is Newton's constant, and the SM Lagrangian density is $L^{\mathcal{G}}_{SM}(x)$. One gets $L^{\mathcal{G}}_{SM}(x)$ from the usual SM Lagrangian density by standard methods that are presented in Refs. [8].

In the SM there are many massive point particles. Are they black holes in our new approach to quantum gravity? To study this question, we follow Feynman, treat spin as an inessential complication [17], and replace $L^{\mathcal{G}}_{SM}(x)$ in (1) with the simplest case for our question, that of a free scalar field, a free

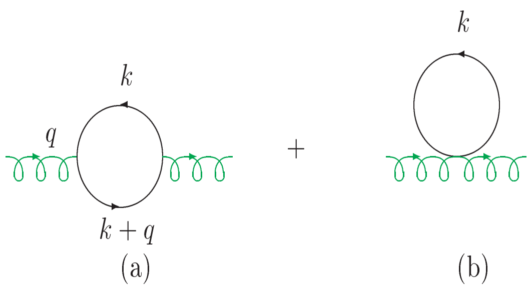

Figure 1. The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

physical Higgs field, $\varphi(x)$, with a rest mass believed to be less than 400 GeV and known to be greater than 114.4 GeV with a 95% CL [18]. We are then led to consider the representative model [9, 10]

$$\begin{aligned}\mathcal{L}(x) &= -\frac{\sqrt{-g}}{2\kappa^2}R + \frac{\sqrt{-g}}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\\ &= \frac{1}{2}\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\\ &\bar{h}_{\mu'\sigma,\sigma'}\} + \frac{1}{2}\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\}\\ &- \kappa h^{\mu\nu}[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}]\\ &- \kappa^2[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2)\\ &- 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}] + \cdots\end{aligned} \tag{2}$$

where $\varphi_{,\mu} \equiv \partial_\mu\varphi$ and we have $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$, $\eta_{\mu\nu} = diag\{1,-1,-1,-1\}$ and $\bar{y}_{\mu\nu} \equiv \frac{1}{2}(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu}y_\rho{}^\rho)$ for any tensor $y_{\mu\nu}$. The Feynman rules for (2) were already worked-out by Feynman [9,10], where we use his gauge, $\partial^\mu\bar{h}_{\nu\mu} = 0$. On this view, quantum gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in Fig. 1. We return to these graphs shortly.

3 Resummed Quantum Gravity

In this section, we will YFS resum the propagators in the theory: from the YFS formula

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'_F(p)}, \tag{3}$$

where $\Sigma'_F(p)$ is the sum of the YFS loop residuals, we need to find for quantum gravity the analogue of

$$\begin{aligned}\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4}\frac{-i\eta^{\mu\nu}}{(\ell^2-\lambda^2+i\epsilon)}\frac{-ie(2ik_\mu)}{(\ell^2-2\ell k+\Delta+i\epsilon)}\\ \frac{-ie(2ik'_\nu)}{(\ell^2-2\ell k'+\Delta'+i\epsilon)}\Big|_{k=k'},\end{aligned} \tag{4}$$

where $\Delta = k^2 - m^2$, $\Delta' = k'^2 - m^2$ and λ is the IR cut-off. With the identifications [19] of the conserved graviton charges via $e \to \kappa k_\rho$ for soft emission from k we get the analogue ,$-B''_g(k)$, of $\alpha B''_\gamma$ by replacing the γ propagator in (4) by the graviton propagator, and by replacing the QED charges by the corresponding gravity charges $\kappa k_{\bar\mu}$, $\kappa k'_{\bar\nu}$. This yields [8]

$$i\Delta'_F(k)|_{Resummed} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)}. \tag{5}$$

with $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right)$ in the deep Euclidean regime. If m vanishes, using the usual $-\mu^2$ normalization point we get $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|k^2|}\right)$. In both cases the resummed propagator falls faster than any power of $|k^2|$! This is the basic result. Note that Σ'_s starts in $\mathcal{O}(\kappa^2)$, so we may drop it in calculating one-loop effects. This means that one-loop corrections are finite! Indeed, all quantum gravity loops are UV finite and the all orders proof, as well as the explicit finiteness of Σ' at one-loop, is given in Refs. [8].

4 Newton's Law

Consider the one-loop corrections to Newton's law implied by the diagrams in Fig. 1. These corrections directly impact our black hole issue. Introducing the YFS resummed propagators into Fig. 1 yields , by the

standard methods [8], that the graviton propagator denominator, $q^2 + \frac{1}{2}q^4\Sigma^{T(2)} + i\epsilon$, is specified by $-\frac{1}{2}\Sigma^{T(2)} \cong \frac{c_2}{360\pi M_{Pl}^2}$ for $c_2 = \int_0^\infty dx x^3(1+x)^{-4-\lambda_c x} \cong 72.1$ where $\lambda_c = \frac{2m^2}{\pi M_{Pl}^2}$. This implies the potential $\Phi_N(r) = -\frac{G_N M_1 M_2}{r}(1-e^{-ar})$ where $a = 1/\sqrt{-\frac{1}{2}\Sigma^{T(2)}} \simeq 3.96 M_{Pl}$ where for definiteness we set $m \cong 120$GeV.

We note for completeness that $c_2 \cong \ln\frac{1}{\lambda_c} - \ln\ln\frac{1}{\lambda_c} - \frac{\ln\ln\frac{1}{\lambda_c}}{\ln\frac{1}{\lambda_c}-\ln\ln\frac{1}{\lambda_c}} - \frac{11}{6}$ and we used this result to check our numerical result for c_2. Without resummation, $\lambda_c = 0$, our result for c_2 would be infinite. Our gauge invariant result for $\Sigma^{T(2)}$ can be shown [8] to be consistent with the one-loop analysis of QG in Ref. [20].

Our deep Euclidean studies are complementary to the low energy studies of Ref. [21]. The effective cut-off which we generate dynamically is at M_{Pl} so that renormalizable quantum field theory (QFT) below M_{Pl} is unaffected. Some non-renormalizable QFT's are given new life here – they may have other problems, however.

5 Massive Elementary Particles and Black Holes

Focusing the previous results, note that ,in the SM, there are now believed to be three massive neutrinos [22], with masses that we estimate at ~ 3 eV, and there are the remaining members of the known three generations of Dirac fermions $\{e,\mu,\tau,u,d,s,c,b,t\}$. With reasonable estimates and measurements [23] of the SM particle masses, including the various bosons, the result for c_2 for each SM massive degree of freedom implies approximately $c_{2,eff} \cong 9.26\times 10^3$ so that in the SM $a_{eff} \cong 0.349 M_{Pl}$. To make direct contact with black hole physics, note that, if r_S is the Schwarzschild radius, for $r \to r_S$, $a_{eff} r \ll 1$ so that $|2\Phi_N(r)|_{m_1=m}/m_2| \ll 1$. This means that $g_{00} \cong 1+2\Phi_N(r)|_{m_1=m}/m_2$ remains positive as we pass through the Schwarzschild radius. It can be shown [8] that this positivity holds to $r=0$. Similarly, g_{rr} remains negative through r_S down to $r=0$ [8]. In resummed QG, a massive point particle is not a black hole.

Our results imply $G_N(k) = G_N/(1+\frac{k^2}{a_{eff}^2})$ which is fixed point behavior for $k^2 \to \infty$, in agreement with the phenomenological asymptotic safety approach of Ref. [13]. Our result that an elementary particle has no horizon also agrees with the result in Ref. [13] that a black hole with a mass less than $M_{cr} \sim M_{Pl}$ has no horizon. The basic physics is the same: $G_N(k)$ vanishes for $k^2 \to \infty$.

Because our value of the coefficient of k^2 in the denominator of $G_N(k)$ agrees with that found by Ref. [13], if we use their prescription for the relationship between k and r in the regime where the lapse function vanishes, we get the same Hawking radiation phenomenology as they do: a very massive black hole evaporates until it reaches a mass $M_{cr} \sim M_{pl}$ at which the Bekenstein-Hawking temperature vanishes, leaving a Planck scale remnant.

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References

1. C. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation*,(Freeman, San Francisco, 1973).
2. S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*,(John Wiley, New York, 1972).
3. R. Adler, M. Bazin and M. Schiffer, *Introduction to General Relativity* ,(McGraw-Hill, New York, 1965).

4. S.L. Glashow, *Nucl. Phys.* **22** (1961) 579; S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264; A. Salam, in *Elementary Particle Theory*, ed. N. Svartholm (Almqvist and Wiksells, Stockholm, 1968), p. 367; G. 't Hooft and M. Veltman, *Nucl. Phys.*B **44**, 189 (1972) and **B50**, 318 (1972); G. 't Hooft, *ibid.* **35**, 167 (1971); M. Veltman, *ibid.* **7**, 637 (1968); D. J. Gross and F. Wilczek, *Phys. Rev. Lett.* **30** (1973) 1343; H. David Politzer, *ibid.***30** (1973) 1346; see also , for example, F. Wilczek, in *Proc. 16th International Symposium on Lepton and Photon Interactions, Ithaca, 1993*, eds. P. Drell and D.L. Rubin (AIP, NY, 1994) p. 593, and references therein.
5. See, for example, M. Green, J. Schwarz and E. Witten, *Superstring Theory, v. 1 and v.2*, (Cambridge Univ. Press, Cambridge, 1987) and references therein.
6. See, for example, J. Polchinski, *String Theory, v. 1 and v. 2*, (Cambridge Univ. Press, Cambridge, 1998), and references therein.
7. See for example V.N. Melnikov, *Gravit. Cosmol.* **9**, 118 (2003); L. Smolin, hep-th/0303185, and references therein.
8. B.F.L. Ward,*Mod. Phys. Lett.*A**17**, 2371 (2002); *ibid.***19** (2004)143; *J. Cos. Astropart. Phys.***0402**, 011 (2004).
9. R. P. Feynman, *Acta Phys. Pol.* **24**, 697 (1963).
10. R. P. Feynman, *Feynman Lectures on Gravitation*, eds. F.B. Moringo and W.G. Wagner (Caltech, Pasadena, 1971).
11. S. Weinberg, in*General Relativity*, eds. S.W. Hawking and W. Israel,(Cambridge Univ. Press, Cambridge, 1979) p.790.
12. O. Lauscher and M. Reuter, hep-th/0205062, and references therein.
13. A. Bonnanno and M. Reuter, *Phys. Rev.* D**62**, 043008 (2000).
14. D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. **13**, 379 (1961); see also K. T. Mahanthappa, *Phys. Rev.* **126**, 329 (1962), for a related analysis.
15. See S. Jadach *et al.*,*Comput. Phys. Commun.* **102**,229 (1997); S. Jadach, M. Skrzypek and B.F.L. Ward,*Phys.Rev.* D**55**,1206 (1997); S. Jadach, B.F.L. Ward and Z. Was,*Phys. Rev.* D**63**, 113009 (2001); S. Jadach, B.F.L. Ward and Z. Was, *Comp. Phys. Commun.* **130**, 260 (2000); S. Jadach *et al.*, *ibid.***140** 432, 475, (2001).
16. S. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).
17. M.L. Goldberger, private communication, 1972.
18. D. Abbaneo *et al.*, hep-ex/0212036; see also, M. Gruenewald, hep-ex/0210003, in *Proc. ICHEP02*, eds. S. Bentvelsen *et al.* (North-Holland, Amsterdam, 2003) 280.
19. S. Weinberg, *The Quantum Theory of Fields, vols. 1-3*, (Cambridge University Press, Cambridge, 1995,1996,2000).
20. G. 't Hooft and M. Veltman, *Ann. Inst. Henri Poincare* **XX**, 69 (1974).
21. See for example J. Donoghue, *Phys. Rev. Lett.* **72**, 2996 (1994); *Phys. Rev.* D**50**, 3874 (1994); preprint gr-qc/9512024; J. Donoghue *et al.*, *Phys. Lett.* B**529**, 132 (2002), and references therein.
22. See for example D. Wark, in *Proc. ICHEP02*, eds. S. Bentvelsen *et al.* (North-Holland, Amsterdam, 2003) 164; M. C. Gonzalez-Garcia, *op. cit.* 186, hep-ph/0211054, and references therein.
23. K. Hagiwara *et al.*, *Phys. Rev.* D**66**, 010001 (2002)1; see also H. Leutwyler and J. Gasser, *Phys. Rept.* **87**, 77 (1982), and references therein.